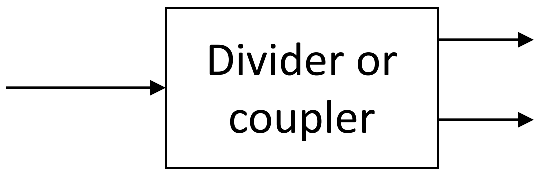
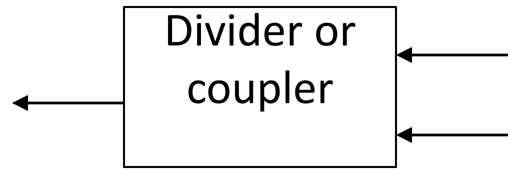
UNIT-2

Microwave Engineering

Power Dividers and Directional Couplers



Power division



Power combining

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \quad , \quad [\mathrm{S}][S]^{*T} = \text{Identity matrix} \Rightarrow$$

$$|\mathrm{S}_{12}|^2 + |S_{13}|^2 = 1 \quad , \quad |\mathrm{S}_{12}|^2 + |S_{23}|^2 = 1 \quad , \quad |\mathrm{S}_{13}|^2 + |S_{23}|^2 = 1$$

$$S_{13}^{*} S_{23} = 0 \quad , \quad S_{23}^{*} S_{12} = 0 \quad , \quad S_{12}^{*} S_{13} = 0$$

$\therefore$ At least two of the three parameters $(\mathrm{S}_{12}, \mathrm{S}_{13}, \mathrm{S}_{23})$ must be zero.

A three port cannot be lossless, reciprocal, and matched at all ports.

Four-Port Network (Directional Couplers)

Assume all ports are matched

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$\text{Lossless} \Rightarrow S_{13}^{*}S_{23} + S_{14}^{*}S_{24} = 0 \;, \quad S_{14}^{*}S_{13} + S_{24}^{*}S_{23} = 0$$

$$S_{14}^{*}\left(\left|S_{13}\right|^{2} - \left|S_{24}\right|^{2}\right) = 0$$

$$S_{12}^{*}S_{23} + S_{14}^{*}S_{34} = 0 \;, \quad S_{14}^{*}S_{12} + S_{34}^{*}S_{23} = 0 \Rightarrow$$

$$S_{23}\left(\left|S_{12}\right|^{2} - \left|S_{34}\right|^{2}\right) = 0$$

$$S_{14} = S_{23} \quad \text{which results in a directional coupler}$$

$$|S_{12}|^2 + |S_{13}|^2 = 1 \ , \ |S_{12}|^2 + |S_{24}|^2 = 1$$

$$|S_{13}|^2 + |S_{34}|^2 = 1 \ , \ |S_{24}|^2 + |S_{34}|^2 = 1$$

$$|S_{13}| = |S_{24}| = \beta \ , \ |S_{12}| = |S_{34}| = \alpha$$

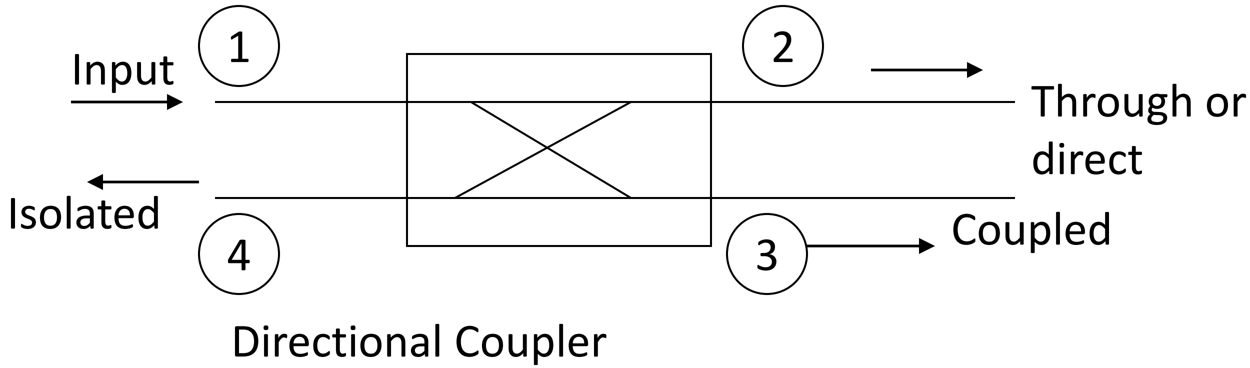
$$S_{12} = S_{34} = \alpha \ , S_{13} = \beta e^{j\theta} \ , S_{24} = \beta e^{j\phi}$$

$$S_{12}^* S_{13} + S_{24}^* S_{34} = 0 \Rightarrow \theta + \phi = \pi \pm 2n\pi$$

$$\alpha^2 + \beta^2 = 1$$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \quad \text{(The symmetrical Coupler)}$$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & \beta \\ \beta & 0 & 0 & \alpha \\ 0 & \beta & \alpha & 0 \end{bmatrix} \quad \text{(The Antisymmetrical Coupler}$$



$$\text{Coupling} = C = 10\log\frac{P_1}{P_3} = -20\log\beta \ \text{dB}$$

$$\text{Directivity} = D = 10\log\frac{P_3}{P_4} = 20\log\frac{\beta}{|S_{14}|}\ \text{dB}$$

$$\text{Isolation} = I = 10\log\frac{P_1}{P_4} = -20\log|S_{14}| \ \text{dB}$$

$$I = D + C \ \text{dB}$$

Hybrid couplers are special cases of Directional coupler, where the coupling factor is 3 dB.

$\alpha = \beta = \frac{1}{\sqrt{2}}$. The quadrature hybrid has a 90^0 phase shift between ports 2 and 3 when fed at port 1.

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

The magic T hybrid or rat - race has a 180^o phase differnce between ports 2 and 3 when fed at port 4.

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$